Gluon penguin enhancements to inclusive charmless decays of $b$ quark in the 2HDM with flavor changing couplings

Zhenjun Xiao $^{a,b,1}$, Chong Sheng Li $^a$, Kuang-Ta Chao $^a$

$^a$ Department of Physics, Peking University, Beijing 100871, PR China
$^b$ Department of Physics, Henan Normal University, Xinxiang 453002, PR China

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Abstract

We calculate the enhancements to the inclusive charmless decays of $b$ quark, $b \rightarrow s q \bar{q}, s q g$, from gluon penguin diagrams induced by the charged and neutral Higgs bosons ($H^\pm, h^0, H^0$, and $A^0$) in the Two-Higgs-Doublet Model with flavor-changing couplings. Within the considered parameter space, the new contributions from charged Higgs boson are dominant. After including the new contributions, the branching ratio $BR(b \rightarrow sg)(q^2 = 0)$ can be increased from $\sim 0.2\%$ in the standard model to $4.4\%$ and $2.6\%$ in the two-Higgs-doublet model for $m_{H^\pm} = 100$ and 200 GeV, respectively. The new contribution to the decay mode $b \rightarrow s q q$ with $q = (u,d,s)$ is, however, numerically small and peaked at the lower $q^2$ region. The new contribution to $b \rightarrow s g g$ can also be neglected.

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Among radiative decays of $B$ meson, $b \rightarrow s g$ is theoretically clean, phenomenologically interesting and sensitive to new physics beyond the standard model (SM), for example, the two-Higgs-doublet model (2HDM) [1], the minimal supersymmetric standard model (MSSM) [2,3] and Technicolor models [4].

In the SM, $BR(b \rightarrow s g) \sim 0.2\%$ for on-shell gluon and $BR(B \rightarrow X_{no \text{charm}}) \sim 1\%$ - 2\% [5]. According to the studies in Ref. [6], an enhanced $b \rightarrow s g$ is favored phenomenologically since such enhancement is very helpful for example to decrease the averaged charm multiplicity $n_c$ and the semileptonic branching ratio [7] and to increase the kaon yields [8]. For the large $BR(B \rightarrow \eta' X_s)$ measured by CLEO [9], one can also give a plausible interpretation from an enhanced $b \rightarrow s g$ [6,10]. The contributions to the ratio $b \rightarrow s g$ in 2HDM without flavor changing (FC) couplings were calculated in Refs. [11,12], and the authors found that this ratio less than 0.7\% for $m_{H^\pm} \geq 200$ GeV and $\tan \beta \sim 5$ in both Type I and II 2HDM [1]. Such an enhancement is not large enough to meet the requirement. The possibility of a large rate for $b \rightarrow s g$ in supersymmetric models were also studied in Ref. [13].

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1 E-mail: zxiao@ibm320h.phy.pku.edu.cn

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In this letter, we calculate the contributions to the inclusive charmless decays $b \to s g$ ($q^2 = 0$), $b \to s q \bar{q}$ and $b \to s g g$ from the gluon penguin diagrams induced by the exchange of charged and neutral Higgs bosons in the so-called Model III: the two-Higgs-doublet model with FC couplings [14,15]. We found that the branching ratio $BR(b \to s g)$ can be increased from $\sim 0.2\%$ in the SM to 2–4\% level in Model III. So large enhancement is still consistent with the CLEO limit: $BR(b \to s g^*) < 6.8\%$ at 90\% C.L. [16], and will be very helpful to resolve the experimental puzzles [6].

In the 2HDM, the tree level flavor changing scalar currents (FCSC’s) are absent if one introduces an ad hoc discrete symmetry to constrain the 2HDM scalar potential and Yukawa Lagrangian. Let’s consider a Yukawa Lagrangian of the form [15]

$$\mathcal{L}_y = \eta^{U}_{ij} \overline{Q}_{i,L} \phi_1 U_{j,R} + \eta^D_{ij} \overline{Q}_{i,L} \phi_1 D_{j,R} + \xi^{U}_{ij} \overline{Q}_{i,L} \phi_2 U_{j,R} + \xi^D_{ij} \overline{Q}_{i,L} \phi_2 D_{j,R} + \text{h.c.},$$

where $\phi_i$ ($i = 1, 2$) are the two Higgs doublets, $\phi^*_{1,2} = \tau_2 \phi_{1,2}$, $Q_{i,L}$ with $i = (1, 2, 3)$ are the left-handed quarks, $U_{j,R}$ and $D_{j,R}$ are the right-handed up- and down-type quarks, while $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ ($i, j = 1, 2, 3$ are family index) are generally the nondiagonal matrices of the Yukawa coupling. By imposing the discrete symmetry ($\phi_1 \to -\phi_1, \phi_2 \to \phi_2, D_i \to -D_i, U_i \to \mp U_i$) one obtains the so called model I and II.

In this letter, we will consider the third type of 2HDM: the so-called Model III [14]: no discrete symmetry is imposed and both up- and down-type quarks then have FC couplings with $\phi_1$ and $\phi_2$. In Model III, there are five standard Higgs bosons: the charged scalar $H^\pm$, the neutral CP even scalars $H^0$ and $h^0$ and the CP odd pseudoscalar $A^0$. After the rotation that diagonalizes the mass matrix of the quark fields, the Yukawa Lagrangian of quarks are the form [15],

$$\mathcal{L}_y^{\text{III}} = \eta^{U}_{ij} \overline{Q}_{i,L} \phi_1 U_{j,R} + \eta^D_{ij} \overline{Q}_{i,L} \phi_1 D_{j,R} + \xi^{U}_{ij} \overline{Q}_{i,L} \phi_2 U_{j,R} + \xi^D_{ij} \overline{Q}_{i,L} \phi_2 D_{j,R} + \text{h.c.},$$

where $\eta^{U,D}_{ij} = m_{ij} \delta_{ij} / v$ correspond to the diagonal mass matrices of quarks and $v \approx 246$ GeV is the vacuum expectation value of $\phi_1$, while the neutral and charged FC couplings will be [15,17]

$$\xi^{U,D}_{\text{neutral}} = \xi^{U,D}_{\text{charged}} = \xi^{U,D}_{v} V_{\text{CKM}},$$

$$\xi^{D}_{\text{charged}} = V_{\text{CKM}} \xi^{D},$$

where $V_{\text{CKM}}$ is the Cabibbo-Kabayashi-Maskawa mixing matrix [18], and

$$\xi^{U,D}_{ij} = \frac{|m_{ij}|}{v} \lambda_{ij}.$$ 

In this letter we assume that $\lambda_{ij}$ are real because we here do not consider the possible effects of CP violation induced by the phase of $\lambda_{ij}$.

As pointed in Ref. [15], the experimental data of $K^0-L\bar{K}^0$ and $B^0_d-\bar{B}^0_d$ mixing processes put severe constraint on the FC couplings involving the first generation of quarks [15]: $(\lambda_{11}, \lambda_{21}) \ll 1$ for $i,j = 1,2,3$. We here will enforce the same constraint: $\lambda_{ii,d} = 0$ for $i,j = 1,2,3$. And we also assume that $\lambda_{ij} = 1$ for $i,j = 2,3$.

Direct searches for Higgs bosons in 2HDM at LEP II [19] place the following mass limits: $M_{H^\pm} > 56$ GeV, $M_{h^0} > 77$ GeV, $M_{A^0} > 78$ GeV. From the CLEO data of $BR(B \to X_s \gamma)$, some constraint on $M_{H^\pm}$ in Model III can also be derived [20,21]. If one uses new CLEO result [22] of $BR(B \to X_s \gamma) \leq 4.77 \times 10^{-4}$ at the $3\sigma$ level, the constraint $M_{H^\pm} \geq 400$ GeV can be read off directly from Fig. 2 of Ref. [20]. According to the studies in Ref. [21], the existence of a charged Higgs boson with $M_{H^\pm} \sim 200$ GeV is still allowed. In this letter, therefore, we will consider the mass range of 100 GeV to 800 GeV for all Higgs bosons in Model III.

As for the additional constraint on the Higgs boson masses from some other processes [15,21], they are not good enough to compete with the $F^0-\bar{F}^0$ mixing processes.

In the SM, the magnetic-penguin induced $b \to s g$ coupling leads to $b \to s g^*$ transitions where $g^*$ could be light-like ($q^2 = 0$, on-shell gluon), or time-like ($q^2 > 0$, off-shell gluon). Under the spectator approximation, there are basically three types of
subprocesses: $b \rightarrow s g$, $b \rightarrow s g q$ for $q = u, d, s$, and $b \rightarrow s g g$. It is straightforward to find the effective $bsg$ coupling by explicit calculation, or by making appropriate changes to the sdy vertex of Ref. [23],

$$
\Gamma_\mu(q^2) = \frac{i g_\mu}{4 \pi^2} \hat{u}_i(p) T^a \nu_y(p^2) u_j(p)
$$

(5)

with

$$
V_\mu(q^2) = \frac{g_\mu^2}{8 M_W^2} \left\{ \left( q^2 s_{\mu \nu} - p_{\mu} q_{\nu} \right) \gamma^\nu \right. 
\times \left[ F_1^L(q^2) L + F_2^R(q^2) R \right] 
+ i q_{\mu \nu} q^0 \left[ m_i F_1^L(q^2) L + m_b F_2^R(q^2) R \right] \right\},
$$

(6)

where $p$ ($p_i$) is the $b$ ($s$) quark momentum, $q = p - p_i$ is the gluon momentum, $F_1^L$ and $F_2^R$ are the electric and magnetic form factors, $g_s$ ($g_\mu$) is the QCD (electroweak) coupling constant, $L, R = (1 \mp \gamma_5)/2$ are the chirality projection operators and $T^a$ with $a = 1, \ldots, 8$ are the $SU(3)_C$ generators.

If the heavy top quark is the internal quark in the penguin diagram, the terms proportional to $m_t^2/M_W^2, m_s^2/M_W^2, q^2/m_t^2$ can be neglected and we then have [23,24]

$$
F_1^L(0) = \sum_i f_i(x_i), \quad F_1^R(0) = 0,
$$

(7)

$$
F_2^L(0) = F_2^R(0) = \sum_i v_i f_2(x_i),
$$

(8)

with

$$
f_i(x_i) = \frac{1}{12(1 - x_i)^2} \left[ 18 x_i - 29 x_i^2 + 10 x_i^3 + x_i^4 
- (8 - 32 x_i + 18 x_i^2) \ln x_i \right],
$$

(9)

$$
f_2(x_i) = \frac{-x_i}{4(1 - x_i)^2} \left[ 2 + 3 x_i - 6 x_i^2 + x_i^3 + 6 x_i \ln x_i \right],
$$

(10)

where $x_i = m_i^2/M_W^2$, $v_i = V_{is}^* V_{ib}$ for $i = u, c, t$.

For decay $b \rightarrow s g$, $q^2_{\text{max}} \approx 20 \text{GeV}^2$ and the assumption $q^2 \ll m_t^2$ which would justify the replacement of the form factors with their values at $q^2 = 0$ is valid only for top quark, but not for the light $u$ and $c$ quarks [3,25]. The correct $f_i(x_i)$ for $j = u, c$ have been given for example in Ref. [3], and we also calculated the $f_2(x_i)$ for $j = (u, c)$ by using the same technique as Refs. [3,25],

$$
f_i(x_i, q^2) = \frac{10}{3} - \frac{2}{3} x_i \ln x_i + \frac{2}{3 z_i} - \frac{2(1 + 2 z_i)}{z_i} g(z_i),
$$

(11)

$$
f_2(x_i, q^2) = -x_i \left( \ln x_i + 2 g(z_i) \right),
$$

(12)

where $z_i = q^2/(4m_i^2)$, and the explicit expression of $g(z_i)$ can be found for example in Ref. [3]. For $q^2 > 4m_s^2$, the internal $u$ or $c$ quark are on mass-shell, an absorptive part therefore appears for $f_1(x_i)$ and $f_2(x_i)$, respectively.

In Model III, an effective $bsg$ coupling can also be induced by the gluon penguin diagram via exchanges of charged and neutral Higgs bosons ($H^0, h^0, A^0, H^\pm$) as depicted in Fig. 1. We will evaluate the penguin diagrams involving $H^\pm$ and then extend the calculation to the neutral Higgs bosons. We will follow the same procedure as that in the SM. We will use dimensional regularization to regulate all the ultraviolet divergence in the virtual loop corrections and adopt the MS renormalization scheme. The necessary Feynman rules can be obtained from the Lagrangian in Eq. (2).

The Feynman diagrams with light internal $u$ and $d$ quarks do not contribute since we have assumed that $\lambda_{u, d, j} = 0$ for $i, j = (1, 2, 3)$. By explicit analyti-

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1}
\caption{Self-energy and gluon penguin diagrams with charged and neutral scalar or pseudoscalar exchanges in Model III. The charged and neutral Higgs boson propagators correspond to the up-type quarks $u_j = (u, c, t)$ and the down-type quarks $d_j = (d, s, b)$, respectively.}
\end{figure}
cal calculations, we can extract out the form factors $F_{1,2}^{L,R}$ and $F_{1,2}^{L,R}$ which describe the new contributions from the neutral and charged Higgs bosons.

For the case of charged Higgs boson $H^\pm$, the effective $bsg$ vertex is of the form,

$$ \Gamma_\mu(q^2) = \frac{ig_s}{4\pi^2} \frac{g_2^2}{8M_W^2} a_i(p_s) T^a \times \left( (q^2 g s_{\mu\nu} - q_{\mu} q_{\nu}) \gamma^\nu \left[ \tilde{F}_{1}^R \tilde{L} + \tilde{F}_{1}^R \tilde{R} \right] 
+ i\sigma_{\mu\nu} q^\nu \left[ m_b \tilde{F}_{1}^L + m_b \tilde{F}_{1}^R \right] \right) \times u_b(p_s), $$

where

$$ \tilde{F}_{1}^R = \frac{1}{M_{H^\pm}} \left[ C_1(x_i) - C_{11}(x_i) \right] \tilde{B}_i, $$

$$ \tilde{F}_{1}^L = \frac{1}{M_{H^\pm}} \left[ 3C_1(x_i) \left[ \frac{m_b}{m_s} \tilde{A}_i + \tilde{B}_i \right] 
- C_1(x_i) \left[ \frac{m_b}{m_s} \tilde{A}_i + \tilde{B}_i + \frac{2m_i}{m_s} \tilde{D}_i \right] \right], $$

$$ \tilde{F}_{2}^R = \frac{1}{M_{H^\pm}} \left[ 3C_1(x_i) \left[ \frac{m_b}{m_s} \tilde{A}_i + \tilde{B}_i \right] 
- C_1(x_i) \left[ \frac{m_b}{m_s} \tilde{A}_i + \tilde{B}_i + \frac{2m_i}{m_s} \tilde{D}_i \right] \right], $$

with

$$ C_1(x_i) = \frac{3 - x_i}{4(1 - x_i)^2} + \frac{1}{2(1 - x_i)^3} \log x_i, $$

$$ C_{11}(x_i) = \frac{11 - 7x_i + 2x_i^2}{36(1 - x_i)^2} + \frac{1}{6(1 - x_i)^3} \log x_i, $$

$$ \tilde{A}_i = \frac{e^2}{2} (V_{CKM} \xi^D)^j_i (V_{CKM} \xi^D)_{jb}, $$

$$ \tilde{B}_i = \frac{e^2}{2} (\xi^U V_{CKM})^j_i (\xi^U V_{CKM})_{jb}, $$

$$ \tilde{C}_i = -\frac{e^2}{2} (V_{CKM} \xi^D)^j_i (\xi^U V_{CKM})_{jb}, $$

$$ \tilde{D}_i = -\frac{e^2}{2} (\xi^U V_{CKM})^j_i (\xi^U V_{CKM})_{jb}, $$

where $\xi^{U,D}$ have been given in Eq. (4), $x_i = m_i^2/M_{H^\pm}^2$. For $i = c, t$. The expressions of $C_i(x_i)$ and $C_{11}(x_i)$ are exact for the case of heavy internal top quark, and correct approximately (the error is less than 5%) for the case of internal c quark.

By the same procedure, we get the effective $bsg$ vertex induced by the neutral Higgs bosons,

$$ \Gamma_\mu(q^2) = \frac{ig_s}{4\pi^2} \frac{g_2^2}{8M_W^2} \sum a_i(p_s) T^a \times \left[ F_i(q^2 g s_{\mu\nu} - q_{\mu} q_{\nu}) \gamma^\nu + iF_2 q_{\mu} q_{\nu} \right] \times u_b(p_s), $$

where the summation over all three kinds of neutral Higgs bosons is understood.

For the CP even scalar Higgs boson $h^0$, we have

$$ F_{1}^{h^0} = \frac{1}{4M_{h^0}^2} \sum_{i = s, b} \left[ 3C_{11}(x_i) - C_1(x_i) \right] \times \left[ \sqrt{m_b} m_{\lambda_i} \lambda_i \sin \alpha \cos \alpha \right], $$

$$ F_{2}^{h^0} = \frac{1}{4M_{h^0}^2} \sum_{i = s, b} \left[ -m_b (3C_{11}(x_i) - C_1(x_i)) 
- 2m_i C_1(x_i) \left[ \sqrt{m_b} m_i \lambda_i \lambda_i \cos^2 \alpha \right] 
- \sqrt{m_b} m_b \lambda_b \sin \alpha \cos \alpha \right], $$

where $x_i = m_i^2/M_{h^0}^2$. 
For the CP even scalar Higgs boson $H^0$, we have

$$F_i^{h^0} = \frac{1}{4M_{h^0}^2} \sum_{i=a,b} \left[ 3C_{11}(x_i) - C_i(x_i) \right] \times \left[ m_{i} m_{\bar{i}} m_1 \lambda_1 \lambda_2 \sin^2 \alpha \right]$$

$$+ \left[ m_{i} m_{\bar{i}} m_2 \lambda_3 \lambda_2 \sin \alpha \cos \alpha \right].$$

$$F_i^{h^0} = \frac{1}{4M_{h^0}^2} \sum_{i=a,b} \left[ -m_h(3C_{11}(x_i) - C_i(x_i)) \right]$$

$$- 2m_i C_i(x_i) \times \left[ m_{i} m_{\bar{i}} m_1 \lambda_1 \lambda_2 \sin^2 \alpha \right]$$

$$+ \left[ m_{i} m_{\bar{i}} m_2 \lambda_3 \lambda_2 \sin \alpha \cos \alpha \right].$$

where $x_i = m_i^2 / M_{h^0}^2$.

For the CP odd pseudoscalar Higgs boson $A^0$, we have

$$F_i^{h^0} = \frac{1}{4M_{h^0}^2} \sum_{i=a,b} \left[ 3C_{11}(x_i) - C_i(x_i) \right] \times \left[ m_{i} m_{\bar{i}} m_1 \lambda_1 \lambda_2 \right].$$

$$F_i^{h^0} = \frac{1}{4M_{h^0}^2} \sum_{i=a,b} \left[ -m_h(3C_{11}(x_i) - C_i(x_i)) \right]$$

$$- 2m_i C_i(x_i) \times \left[ m_{i} m_{\bar{i}} m_1 \lambda_1 \lambda_2 \right].$$

where $x_i = m_i^2 / M_{h^0}^2$, and the functions $C_i(x_i)$ and $C_{11}(x_i)$ in Eqs. (22)–(27) have been given in Eqs. (18) and (19).

In the numerical calculations, we fix the following parameters and use them as the standard input (SIP) [26,27]: $M_W = 80.41$ GeV, $\alpha_{em} = 1/129$, $\sin^2 \theta_W = 0.23$, $m_t = 5$ MeV, $m_t = 9$ MeV, $m_t = 1.4$ GeV, $m_b = 0.13$ GeV, $m_b = 4.4$ GeV, $m_b = 170$ GeV, $A_{1/2}^{(s)} = 0.225$ GeV, $A = 0.84$, $\lambda = 0.22$, $\beta = 0$, and $\eta = 0.36$. For the definitions and values of these input parameters, one can see Refs. [26,27]. For the running of $\alpha_s(\mu)$, we use the two-loop formulae as given in Ref. [27] and take $\mu = m_b$.

Using the SIP and assuming $m_{Higgs} = 300$ GeV and $q^2 = (1-19)$ GeV$^2$, we find numerically that:

- In the SM, $|F_i^{L,R} = 0.25$, $|F_i^{L,R} = 0.008$; For all neutral Higgs bosons, $F_i$ and $F_2$ form factors are less than $10^{-4}$ in magnitude and therefore can be neglected safely;

- For charged Higgs boson, $F_i^{L,R}$ are very small and therefore can be neglected, but $F_i^{L,R}$ is larger than its SM counterpart by about two orders of magnitude. While the $F_i^{L,R} = 0.009$ is comparable with $F_i^{L,R}$ in the SM.

In the following, we will only consider the new effects of $F_i^{L,R}$ form factors induced by charged Higgs boson. The $q^2$ dependence of $F_i$ and $F_2$ in the SM and the $F_i^{L,R}$ in Model III comes from the penguin diagrams with $c$ internal quarks. But this dependence is very weak and therefore can be neglected safely.

Now we turn to calculate the inclusive charmless decay $b \to sg$ for both on-shell and off-shell ($q^2 > 0$) gluon, in order to check the possible enhancements to the decay widths and branching ratios induced by the charged Higgs boson in Model III.

Fig. 2. Feynman diagrams for the charmless decay of $b$ quark in Model III: (a) $b \to sg$ ($q^2 = 0$, on-shell gluon); (b) $b \to sg$ ($q^2 > 0$) with $q = u,d,s,c$; (c) additional diagram for $b \to sgg$ via $g \to gg$; (d) typical one-particle reducible diagram that lead to $b \to sgg$; (e) typical one-particle irreducible diagram that lead to $b \to sgg$. The blobs in first five diagrams denote the effective $bsg$ vertex.
For the decay $b \to sg$ with an on-shell gluon as depicted in Fig. 2a, it is easy to derive the decay rate

$$\Gamma(b \to sg)^{\text{SM}} = \frac{2\alpha_s(m_b)}{\pi} |F_2|_0^2$$

in the SM, and

$$\Gamma(b \to sg)^{\text{III}} = \frac{2\alpha_s(m_b)}{\pi} \Gamma(\frac{F_2^{R,R} + \frac{m_b}{m_s} F_2^{L}}{F_2^{L}})$$

in Model III. Here $\Gamma_0 = G_F^2 m_b^5/(192\pi^3)$, $F_2^{R,R} = F_2 + \frac{m_b}{m_s} F_2^{L}$, the SM form factor $F_2$ has been given in Eq. (8). For $M_{H^+} = 200$ GeV, one finds

$$R_g = \frac{\Gamma(b \to sg)^{\text{III}}}{\Gamma(b \to sg)^{\text{SM}}} = 14.6$$

and the mass dependence of $R_g$ is shown in Fig. 3a.

The branching ratio $BR(b \to sg)$ is of the form

$$BR(b \to sg) = \frac{3\alpha_s(m_b)}{\pi |V_{cb}^2| f(z) \kappa(z)} \left\{ \begin{array}{ll}
|F_2|^2, & \text{in SM,} \\
|F_2^{R,R}|^2 + \frac{m_b}{m_s} |F_2^{L}|^2, & \text{in Model III,}
\end{array} \right.$$  

where $f(z) = 0.54$ is the phase space factor in the semileptonic $b$-decay with $z = m_c/m_b$, $\kappa(z) = 0.88$ is the QCD factor to the semileptonic $b$-decay, and $BR(B \to X_u e\bar{\nu}_e) = 10.5\%$ [26].

Figs. 3a and 3b are the plots of $R_g$ and the ratio $BR(b \to sg)$ versus $M_{H^+}$ in all three types of 2HDM for $M_{H^+} = (100–800)$ GeV. For theoretical predictions in Model I and Model II, we directly used the formulae given in Ref. [11]. For $M_{H^+} = 200$ GeV, the branching ratio $BR(b \to sg)$ will be increased from $\sim 0.2%$ in the SM to $\sim 2.6%$ in Model III, as shown by the upper solid curve in Fig. 3b. The short-dashed and dotted curve in Figs. 3a and 3b shows the ratios in Model I and Model II, respectively. It is easy to see that the enhancements to the ratios in both Model I and II are much smaller than that in Model III.

For $q^2 > 0$, the gluon is virtual and time-like, therefore before it can fragment into real hadrons, it first disintegrates into real partons such as on-shell quark pair $q\bar{q}$ for $q = (u, d, s)$ and gluon pair $gg$, as shown in Fig. 2. The process $b \to u\bar{u}$ and $b \to d\bar{d}$ can be treated on the same footing, while one should take into account the identical particle effects for the decay $b \to s\bar{s}$. Here, as an illustration, we will evaluate the new contributions to the branching ratio $BR(b \to sq\bar{q})$ for $q = u, d, s$ in Model III. We use the same method as Ref. [24].
In order to illustrate the effects of charge Higgs boson on the differential decay rate, we draw the Fig. 4 for the typical decay mode $b \to s \bar{u} \bar{u}$.

$$\frac{d\Gamma(b \to s \bar{u} \bar{u})}{dy} = \frac{\alpha_s^2(m_b)}{6\pi^2} \Gamma_0 \left( (1-y)^2(1+2y)|F_1|^2 + \frac{1}{y} \left[ 2 - 3y + y^3 \right] \left[ |F_2|^2 + \frac{m_s^2}{m_b^2} |F_2|^2 \right] - 6(1-y)^2 \text{Re}\left[ (F_1^* F_2) \right] + \frac{6m_s^2}{m_b^2} (1-y^2) \text{Re} \left[ (F_1^* F_2^\pi) \right] - \frac{12m_s^2}{m_b^2} (1-y) \text{Re} \left[ (F_2^\pi F_2^\pi) \right] \right), \quad (32)$$

where $y = q^2/m_b^2 = 1 - 2E_s/m_b$. If one treats the $E_s$ as the kaon energy, then the differential rate in Eq. (32) can be regarded as the “Kaon-energy” spectrum. In Fig. 4, the $c\bar{c}$ threshold cusp is clearly exhibited, where the short-dashed (solid) curve shows the differential rate in the SM (Model III). It is easy to see that the new contribution is peaked at the lower $q^2$ region.

The decay width $\Gamma(b \to sq\bar{q})$ with $q = (u,d,s)$ is of the form

$$\Gamma(b \to sq\bar{q}) = \frac{\alpha_s^2(m_b)}{12\pi^2} \frac{17}{16} |F_1|^2 - \frac{3}{32} \frac{y}{\pi} \text{Re}\left[ (F_1^* F_2) \right]$$

$$- \left( \frac{y}{\pi} + 10 \log[y_{\min}] + 2 \log[x_{\min}] \right)$$

$$\times \left[ |F_2|^2 + \frac{m_s^2}{m_b^2} |F_2|^2 \right]$$

$$+ \frac{24m_s^2}{m_b^2} \text{Re}\left[ (F_1^* F_2^\pi) \right]$$

$$- \frac{30m_s^2}{m_b^2} \text{Re}\left[ (F_2^\pi F_2^\pi) \right], \quad (33)$$

where $x_{\min} = y_{\min} = 4m_b^2/m_s^2$. Contrary to the case of decay $b \to sg$ with on-shell gluon, the new contribution from charged Higgs boson in Model III tend to decrease the decay width $\Gamma(b \to sq\bar{q})$ slightly by about five percent with respect to the SM prediction. The reason for this behaviour is simple. We know that only the new magnetic $F_2$ form factor contribute effectively to the decay processes under study. For the decay $b \to sg$ with on-shell gluon, the magnetic form factor $F_2$ dominate the total contribution. But for the case of $q^2 > 0$, the SM form factor $F_1^L$ control the decay processes, the new $F_2$ can contribute only through interference with the SM $F_1^L$.

The large infrared (IR) logarithms in Eq. (33) will be canceled if one make a complete $O(\alpha_s)$ treatment of the second term in Eq. (33). Since we are not giving a full $O(\alpha_s)$ QCD analysis, we can not include this term consistently. We therefore simply drop it from further discussions, as done in Ref. [24].

The decay mode $b \to sgg$ via or not via $g^* \to gg$ has been studied in Ref. [24]. It was found that the decay width $\Gamma(b \to sgg)$ depend on the form factor $F_1$ only,

$$\Gamma(b \to sgg) = \frac{\alpha_s^2(m_b)}{4\pi^2} |F_1|^2. \quad (34)$$

As discussed previously, the new contributions to the electric form factor $F_1$ from charged and neutral Higgs bosons are very small and has been neglected. The decay $b \to sgg$ therefore will be not affected.
effectively by gluon penguins induced by Higgs bosons appeared in Model III.

Collectively, the branching ratio \( BR(b \rightarrow sg^*) \) (here \( b \rightarrow sg^* \) is symbolic for processes of \( b \rightarrow sg \) (on-shell gluon), \( b \rightarrow s\bar{q}q \), and \( b \rightarrow sgg \)) can be written in the form

\[
BR(b \rightarrow sg^*)^{\text{SM}} = \frac{0.105}{|V_{cb}|^2 f(z) \kappa(z)} \left\{ \frac{3 \alpha_s(m_b)}{\pi} |F_2|^2 + \frac{\alpha_s^2(m_b)}{16\pi^2} \left\{ \frac{35}{\pi} |F_1|^2 - \frac{31}{\pi} \text{Re}\left[ (F_1^* F_2^T) \right] \right\} \right\},
\]

(35)

\[
BR(b \rightarrow sg^*)^{\text{III}} = \frac{0.105}{|V_{cb}|^2 f(z) \kappa(z)} \times \left\{ \frac{3 \alpha_s(m_b)}{\pi} |F_2|^2 + \frac{m_s^2}{m_b^2} |\tilde{F}_2|^2 \right\} + \frac{\alpha_s^2(m_b)}{16\pi^2} \times \left\{ \frac{35}{\pi} |F_1|^2 - \frac{31}{\pi} \text{Re}\left[ (F_1^* F_2^T) \right] \right\} + \frac{24m_s^2}{m_b^2} \text{Re}\left[ (F_1^* F_2^T) \right] - \frac{30m_s^2}{m_b^2} \text{Re}\left[ (\tilde{F}_2^* \tilde{F}_2^T) \right] \right\},
\]

(36)

As illustrated in Fig. 5, the branching ratio \( BR(b \rightarrow sg^*) = 5.5\% - 1.6\% \) for \( M_{H^\pm} = 100 - 800 \) GeV in Model III (solid curve) compared with 1.3% in the SM (short-dashed line). For light charged Higgs, the enhancement is significant: \( BR(b \rightarrow sg^*) = 3.7\% \) for \( M_{H^\pm} = 200 \) GeV in Model III. The upper dot-dashed line in Fig. 5 shows the CLEO upper limit: \( BR(b \rightarrow sg^*) < 6.8\% \) at 90% C.L. [16]. In Ref. [28], the author studied the CP violation in radiative \( B \) decays in a type-III 2HDM with the fourth generation fermions, and found that the branching ratio \( BR(b \rightarrow sg) \) can be increased to 10% level.

To summarize, we have calculated, from the first principle, the new gluon penguin diagrams that contribute to the inclusive charmless decays of \( b \) quark in Model III without inclusion of QCD corrections. We start from the evaluation of the new gluon penguin diagrams induced by the exchange of charged and neutral Higgs bosons \( (H^\pm, h^0, H^0, A^0) \), derive out the \( F_1 \) and \( F_2 \) form factors which control the new contributions to the inclusive \( b \) quark decays under study, and finally calculate the relevant decay rates and branching ratios. We found that:

(a) Among charged and neutral Higgs bosons, the CP even charged Higgs boson \( H^\pm \) dominate the new contribution. By using the assumed FC couplings, the contribution of the neutral scalar and pseudo-scalar is completely negligible. Therefore, both the value of the mixing angle \( \alpha \) and \( m_{H^0}, m_{H^0}, \) and \( m_{A^0} \) are irrelevant.

(b) The new electric form factor \( F_1 \) is also completely negligible. The new magnetic form factor \( \tilde{F}_2 \), on the contrary, is much larger than its SM counterpart and thus contribute significantly to the decay processes in question. The \( \tilde{F}_2^R \) is comparable in size with the \( F_2 \) in the SM and can also contribute effectively.

(c) The charged Higgs enhancement to the branching ratio \( BR(b \rightarrow sg) \) with \( q^2 = 0 \) can be as large as a factor of 25 (14.6) for \( M_{H^\pm} = 100 \) GeV.
(200 GeV) in Model III. So large enhancement will be very helpful to generate a large ratio $BR(b \rightarrow sgg)$ favored by some experimental data such as the deficit in charm counting, a $3\sigma$ deficit in kaon counting as well as the well-known large $B \rightarrow \eta' X$, branching ratio measured by the CLEO collaboration. The enhancement in Model III can be much larger than that in both Models I and II.

(d) For the subprocesses with a off-shell gluon ($q^2 > 0$), the new contribution to the decay mode $b \rightarrow sq\overline{q}$ with $q = (u, d, s)$ is numerically small and peaked at the lower $q^2$ region. And the new contribution to $b \rightarrow sgg$ can be neglected.

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